

Algebra qualifying exam, Spring 2016

1. Classify the groups of order 24 having trivial center.
2. Let V be a finite dimensional vector space over a field of characteristic p and let $T : V \rightarrow V$ be a linear transformation such that $T^p = I$

\bar{d}) for some $d \in \mathbb{Z}$.

5. Suppose n and m are positive integers, let $R = \mathbb{Z}[X]/(X^n)$ and let M be an R -module, let x be the image of X in R , and let (x^m) be the ideal in R generated by x^m . Compute $\text{Tor}_i^R(M, (x^m))$ for all i .
6. Let k be a field. Find the minimal primes and compute the Krull dimension of $k[x, y, z]/(xy, xz)$.
7. Let R be an Artinian local ring. Prove that an R -module is flat if and only if it is free.
8. Suppose that R is a Noetherian ring and \mathfrak{p} is a prime ideal such that $R_{\mathfrak{p}}$ is an integral domain. Show that there is an $S \subseteq R \setminus \mathfrak{p}$ such that R_f is an integral domain. (Recall that $R_f = S^{-1}R$ where $S = \{1, f, f^2, f^3, \dots\}$.)