Fire Sales and Endogenous Volatility

Yue Jiang
Boston University

Draft Version, September 2016

Abstract

After the collapse of the housing bubble in 2007, severe re sales of assets in the nancial sector are accompanied by a rise in the volatility of asset returns in the non- nancial rms. To account for their co-movements, I develop a model that highlights the interaction between the nancial health of the banking sector and the volatility of asset returns. The novel feature of the model is that the volatility of asset returns is endogenously generated by the banks' risk taking behavior. The risk taking by banks imposes a negative externality on the nancial health of other banks because given the risk aversion of secondary market buyers, the liquidation of risky assets depresses the secondary market price of assets. A weak nancial health hurts the bank's long term pro tability. Combining with the limited liability, the model can give rise to a vicious feedback loop between a collective risk taking behavior in the banking sector and re sales of assets. A standard liquidity requirement is shown to have ambiguous e ects in stabilizing the nancial system depending on the asset market liquidity. The model suggests a room for counter-cyclical macro-prudential policy to improve nancial stability.

[1] I am deeply indebted to Christophe Chamley for his constant support and advice during this project. I also bene tted greatly from Dirk Hackbarth, Jianjun Miao, Simon Gilchrist, Stephen Terry and Anton Korinek. Comments are welcome. Email address: yuej@bu.edu.

1 Introduction

The 2007 financial crisis has rekindled the search of the origins of financial fragility. The collapse of the housing bubble in 2007 triggered a liquidity shortage in the banking sector.[1] Banks' financial health deteriorated due to massive losses on assets and withdrawals from short term creditors, which forced banks to liquidate assets in fire sales.[2] Along with the severe fire sales of assets, the economy also experienced a widespread rise in the volatility of asset returns among non-financial firms. Figure 1 documents the evolution of the time-varying volatility of equity returns for non-financial firms[3] and the TED spread[4] in the U.S.. The TED spread indicates the difficulty of the banking sector in financing their long term investments and can be used as a measure of the credit risk. Both variables surged significantly during the recent recession. Moreover, their co-movement seems persistent throughout the past 30 years. To account for these observations, I provide a theoretical explanation that highlights the interaction between the financial health of the banking sector and the volatility of asset returns to explain their co-movement and to explore the potential cause of financial instability.

market. This riskier equilibrium characterizes the economy in financial crises, e.g., the 2007 financial crisis.

The equilibrium with a self-fulfilling crisis generates welfare loss because banks do not internalize their impact on asset prices or the default costs. Therefore, in the second part of the paper I analyze how macro-prudential policies affect the financial market efficiency in my setting. To be more specific, I extend the model by incorporating an ex-ante choice of cash holding and analyze the implication of a liquidity requirement, according to which banks are required to hold certain amount of liquid assets ex ante.

The model suggests that the effect of a liquidity requirement is ambiguous in improving financial stability. When the secondary market price is low, banks are holding liquid assets at a level which is lower than the social optimal level. Liquidity requirement could lower bank's incentive of risk shifting by reducing the credit risk and boosting bank payoffs. When the secondary market price is high, the cost of forfeited long term returns outweighs the benefit from a stronger liquidity buffer. Liquidity holdings reduce long term payoffs, leading to a stronger risk shifting incentive. Therefore, the liquidity requirement poses tradeoff between improving financial stability in economic downturns and encouraging excessive risk taking in economic upturns.

There has been a growing consensus on the implementation of counter-cyclical regulations in promoting the resilience of the financial system. In the context of the model, the counter-cyclical liquidity requirement[8] is shown to improve the tradeoff of a standard liquidity requirement and promote financial stability. The intuition is the following. According to the previous discussion, a higher liquidity requirement could effectively rein in excessive risk taking when fire sales are expected. However, it would not encourage risk taking by much during economic booms because the bank payoffs are less affected. Similarly, a lower liquidity requirement could reduce the risk taking incentives when the secondary market price is high while it may not significantly intensify the risk taking incentive because bank payoffs are already low during economic downturns.

To sum up, I build a model that connects financial health of banks with macroeconomic volatility. An expectation of fire sales can result in a self-fulfilling financial crisis where the risk taking incentives and fire sales reinforce each other. The model suggests a room for counter-cyclical macro-prudential policy to improve financial stability.

The paper is organized as follows. Section 2 discusses research related to the paper. Section 3 studies the basic version of the model, followed by an analysis of comparative statics. Section 4 extends the basic model by incorporating ex ante choice of liquidity and studies the implication of a liquidity requirement. Section 5 concludes. The Appendix

[8] A counter-cyclical liquidity requirement stipulates a higher requirement during economic upturns and a lower requirement in economic downturns.

provides the proofs.

2 Literature Review

The paper connects with several lines of the literature: (i) nancial fragility; (ii) risk taking of banks; (iii) re sales and (iv) time-varying volatility. I discuss how my paper is related to each of the topics and the papers in the intersection of these topics.

First, this paper is related to the literature on the nancial fragility. In the seminal paper by Diamond and Dybvig (1983), the fragility of a nancial system stems from \panics" of depositors on the amount of withdrawals. Along this line of work, Chari and Jagannathan (1988) show that bank runs occur not only when the economic outlook is poor but when liquidity needs are high as well. Allen and Gale (1998) develop a model where panics occur when depositors perceive that the returns on bank assets are going and Weiss (1981) (in a credit market equilibrium context), the risk-shifting phenomenon has been intensively studied.[10] Acharya (2009) develops a model that highlights a systemic risk-shifting incentive that is originated from bank failures. When one bank fails, it exerts negative externality on others by raising the deposit rate. My paper di ers from Acharya(2009) in two aspects. First, my paper does not rely on the actual defaults for the existence of a systemic risk-shifting incentive. Second, instead of choosing the correlation on their long term investments, banks in my model choose the volatility on asset returns. From this perspective, my paper complements Acharya (2009) by looking at the risk from another dimension, with a focus on the volatility in returns.

The way I model the risk taking of banks is similar to Martinez-Miera and Repullo (2015), Repullo(2004), and Navarro (2015). However, the paper focuses on the study of nancial stability, which stems from the bank's expectation of future liquidity shortages and its inability of funding its depositors.

In the empirical front, there are works con rming the existence of the risk shifting incentives. Duran and Lozano-Vivas (2014) examine the risk shifting of US banks in 1998 - 2011. Their results suggest that the risk shifting is present throughout the entire period, with least signi cance for the post crisis period. Moreover, banks engage in risk shifting most signi cantly with non-depository creditors. Landier, Sraer and Thesmar (2011) conduct an interesting case study on the lending behavior of a large subprime mortgage originator - New Century Financial Corporation and show that nancial institutions in distress, may take excessive risk. The incentive distortion e ect of nancial distress and gamble \for resurrection" of banks are intensively studied in Esty (1997), Gan(2004) and Fischer et al (2011). My paper provides theoretical explanation for these ndings that banks in distress tend to engage in risk-shifting and the risk is mostly shifted to un-insured creditors.

Third, the paper is related to the literature on re sales[11]. In light of the recent nancial crisis, people have been focusing on the deterioration of balance sheets of banks and the disruptions of the so-called bank lending channel (See Bernanke and Blinder (1988)). Papers exploring the importance of nancial intermediations include Gertler and Karadi (2011), Gertler and Kiyotaki (2009), Brunnermeier and Pedersen (2009), and Brunnermeier and Sannikov (2011).

Most papers in the literature focus on the contraction of credit supplies due to lower net worths of banks. Departing from the literature, this paper provides a new channel through which re sales causes disruptions in the nancial system by emphasizing the interaction between res sales and the risk taking by banks.

[10]See Bhattacharya et al. (1998) and Freixas and Rochet (2008) for surveys on risk shifting.

[11]See Shleifer and Vishny (2011) for a survey on re sales

Last, the paper is related to the literature on time-varying uncertainty. The seminal paper by Bloom(2009) points out that the time varying volatility can undermine the real economy.[12] Because of the detrimental e ect of time-varying volatilities on the real economy, it is worth exploring where the volatility comes from.

There is a growing line of work that studies endogenous volatilities. A seminal paper on the study of endogenous volatility is Veldkamp (2005) where uncertainty is generated by learning about economic fundamentals.[13] Bachmann and Bayer (2013) use a long panel of German rms and show that shocks to the variance of rm-level TFP innovations, if any, only mildly amplify rst-moment aggregate shocks. The volatility in TFP is not an independent source of aggregate uctuations.[14] Bachmann and Moscarini(2012) explore the reverse causality where negative rst moment shocks induce risky behavior, leading to a rise in volatility in economic outcomes.

Following this lead, I endogenize the volatility of asset returns by relating it with the risk taking by banks. Establishing the link between the volatility and the nancial health of banks, the paper highlights the negative impact of volatility on the nancial system and the real economy.

3 The Model

The model adopts the framework of Diamond-Dybvig Banking model[15]. There are three dates ($t = 0; 1; 2$). The key actors in the model are banks and depositors. Departing from the Diamond-Dybvig model, the paper focuses on the choice of risk by banks and incorporates a secondary market for the long term assets at date 1.

The main subject of study is the bank. However, the mechanism discussed in the paper is not limited to the nancial ins c7.55-44n6(date)-326(1.a326(iscu16 c7.55-44n6(damaio27(

Figure 2: Time Line of the Model

return of the assets. At date 1, when its depositors demand funds, the bank needs to liquidate its long term assets in a secondary market to ful ll their needs. At date 2, long term assets pays o . Given limited liability, the bank has the option to default when its payo is negative.

Long Term Assets Each bank invests in projects that yield returns at date 2. The projects have mean return z, which is realized at date 0. Because the bank makes decisions after the realization of z, in the basic model, z is treated as a parameter. Given the mean return z, the long term project generates random returns z_2 at date 2 according to

$$z_2(z; s; \ ) = (1 + s \ )z. \tag{1}$$

In this equation, s is an idiosyncratic productivity shock at date 2,

$$s = \begin{cases} 1 & \text{with probability } \frac{1}{2}; \\ \ \ 1 & \text{o.w.} \end{cases} \tag{2}$$

represents the riskiness of returns. It is an endogenous choice of the bank. There is a menu of projects available for banks. These projects have the same mean returns but they di er in terms of riskiness. 2 [0;]. For simplicity, I assume = 1. The bank chooses risk by investing in a speci c project.

Banks There is a measure one of ex ante identical banks indexed by i. i 2 [0; 1]. At date 0, each bank attracts one unit of deposits from households and uses the funds to invest in long term assets. In the basic model, banks cannot hold liquid assets ex ante.[16] The deposits are uninsured and generate return R at date 2.[17]

[16] Later on, the paper allows banks to invest in liquid assets that yield risk free returns the next period.

[17] In reality, deposits are insured or regulated in many countries. In the U.S, FDIC has been created in 1933 to provide deposit insurance to depositors in US banks. Apart from commercial banks that are FDIC-insured, there are other non-FDIC-insured nancial corporations, such as investment banks and

Given the mean return of long term assets z, the bank chooses to take risk on its portfolio. There is a cost of risk taking ( ) for each unit of return. Intuitively, the cost can be interpreted as the resources spent for to monitor and collect the realized returns. Assume that ( ) = c , where $c < \frac{1}{2}$.[18]

At date 1, a common liquidity shock[19] hits all banks with probability , in which a fraction x of their depositors withdraw. x is the realization of a random variable drawn from an uniform distribution on [0; 1]. x is private information to the bank and it is not observable to its depositors.

Each bank needs to o er x liquid assets to satisfy the early withdrawals. At the same time, the bank o ers late depositors a return R at date 2 so that they have no incentive to withdraw at date 1.

In order to pay the early withdrawals, the bank has to liquidate its long term assets in the secondary market at price p. p is the price facing all selling banks. In the basic model without ex ante holding of liquid assets, p is also the market value of bank assets at date 1. If $p > x$, the bank is ne. If $p < x$, the bank cannot cover the withdrawals even by selling all long term assets. In this case, the bank is forced to default. Let

$$(p) = \min\{p; 1\}: \qquad (3)$$

If a shock takes place, 1 (p) is the probability that a bank faces withdrawal x greater than p. (1 (p)) is the probability (in date 0) of default (in date 1). Call it the 'illiquidity risk' of the bank.

In this model, the instability of the nancial system stems from the early withdrawal shocks or more precisely, the expectation of the shock. In expectation of an early withdrawal shock, the bank chooses risk to maximize its long term payo . The aggregate riskiness of asset returns determines the asset price in secondary market, which in turn a ects the risk decision of banks through its impact on the illiquidity risk and the credit risk of the bank.

At date 2, the productivity shock s is realized. The long term assets pay o accordingly. The ex post payo of the bank is

$$y(z; x; s;\ ) = (1 \quad \frac{x}{p})(1 \quad (\ ))(1 + s\ )z \quad (1 \quad x)R\ : \qquad (4)$$

With limited liability, a bank will default whenever its payo is negative. When default,

funds, that nance their long term investments with short term debts. The model is more relevant to this type of nancial institutions.

[18] The assumption on c guarantees that the per unit return ex ante with risk is greater than the return with no risk.

[19] The same type of shock has also been used in Diamond and Rajan (2011).

the bank will get zero payo .

Depositors Each bank has a measure 1 of ex ante identical depositors. At date 1, with probability , x fraction of depositors become the early type, which need to withdraw funds immediately. The return for early withdrawals is 1. (1 x) depositors become the late types, who have no needs for funds at date 1.

The late type depositors have the option to withdraw fund together with the early types and invest in safe and liquid assets which yield risk-free return r at date 2. r is exogenous. If the late types do not withdraw, they will get long term return R at date 2 given that the bank does not default. Otherwise, they will get 0 when the bank defaults.[20] The mean return of long term asset is observable to the depositors. But they cannot observe the size of the early withdrawals x or the risk behavior of their individual banks. Given z, depositors would demand a long term rate R such that their expected return for not withdrawing is no less than their outside option r.

Secondary Market A secondary market for the long term assets is opened at date 1. Banks sell their long term assets for liquidity in the market at price p. There are unlimited number of potential buyers for the assets. Buyers are risk averse.

It will be shown later that the bank's choice of risk takes a corner solution, 2 f 0; 1g. So the assets sold in the secondary market would yield either highly risky returns or riskless returns. There is asymmetric information between buyers and banks. Buyers cannot observe the risk associated with a speci c asset. They know only that n^D fraction of assets have risky returns in the market. The risk averse buyers would demand a discount in price for holding the risky assets.

Therefore the market price p for the valuation of asset is given by

$$p^D(n^D; z) = \frac{z}{r}\; n^D(1 \quad) + (1 \quad n^D)\;: \tag{5}$$

This section focuses on the basic version of the model. Later on, the model incorporates an ex ante choice of liquidity and capital respectively. The extensions of the model not only provides a more complete picture of the behavior of banks but it could generate more room for policy analysis as well.

[20] In the appendix, I relax this assumption. In stead of zero payo when default, depositors can get a fraction of the gross return of the bank. Relaxing this assumption does not generate qualitatively di erent results.

3.1 Bank's Choice of Risk

The key decision the bank has to make is the choice of risk, which takes place at date 0. At date 0, the aggregate return on long term assets z

PROPOSITION 1 (Optimal Choice of Risk) Given the price of long term asset in the secondary market p and equilibrium long term deposit rate R, there exists a threshold $z\ (p;R)$ such that

$$(z;p;R) = \begin{cases} 1 & \text{if } z < z\ (p;R) \\ 0 & \text{if } z \quad z\ (p;R): \end{cases}$$

The threshold $z\ (p;R)$ is the following:

$$z\ (p;R) = \ (p)R \tag{7}$$

where (p) is a function of secondary market price p and model parameters.

Proof: (In the Appendix).

The cost and benefit of risk taking differ according to different levels of z.

Figure 3 illustrates the intuition graphically. Without the cost of risk taking, risk taking is always preferred, as shown in the red dotted line. The bank obtains returns only in positive shocks when taking risk $= 1$, so the benefit of risk taking comes from the higher survival probability (in positive shocks) and a lower expected repayment to late depositors.

The cost of risk taking is proportional to the returns on the long term assets. Higher mean return entails a higher cost of risk taking, as shown in the red solid line. After taking into account the cost, there exists a maximum mean return z, beyond which the bank prefer no risk in returns.

Because the bank's expected payoffs with and without risk are both homogenous of degree one in their arguments, an increase in R leads to an one-to-one increase in the threshold return z. The multiple (p) is the ratio of return required to induce stable returns and the borrowing cost R. Intuitively, it is the liquidity premium the bank demands for exposing itself to liquidity shocks while not taking risk.

Proposition 2: (p) is increasing in p for $p < \bar{p}$ and decreasing in p for $p \quad \bar{p}$, where $\bar{p} < 1$ and satisfies that

$$(1 \quad c)x^H(\bar{p};1;\ (\bar{p}))^2 = x^L(\bar{p};1;\ (\bar{p}))^2; \tag{8}$$

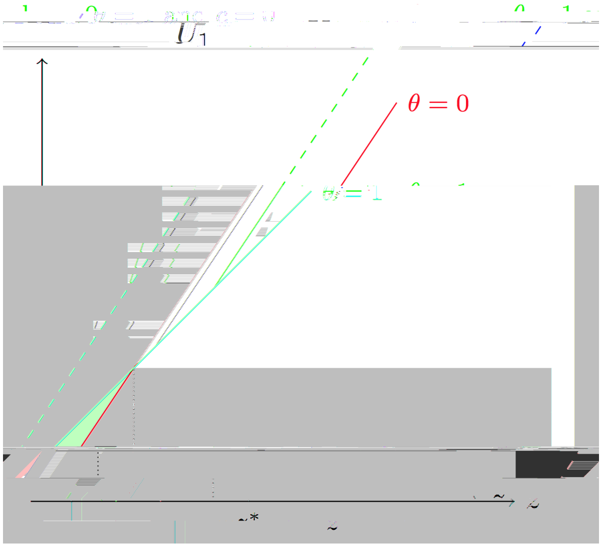

Figure 3: Threshold z

where x^H and x^L are the maximum sizes of early withdrawals that would not induce a default at date 2, when the bank take high risk (H) and no risk (L),

$$x^H(p; R ; z) = \frac{z \quad \frac{R}{2(1 \quad c)}}{\frac{z}{p} \quad \frac{R}{2(1 \quad c)}} \tag{9}$$

and

$$x^L(p; R ; z) = \frac{z \quad R}{\frac{z}{p} \quad R}: \tag{10}$$

Proof: (In the Appendix).

(p) is non-monotonic in p. It rst increases and then decreases in p. The intuition is the following.

For p su ciently low, i.e., p < p, the secondary market is almost completely illiquid. Regardless of the choice of risk, the bank cannot sell long term assets for liquidity and is forced to default when hit by a liquidity shock. The bank can only generate positive payo s when facing no early withdrawals. Without liquidity shocks, the bank has less incentive to take risk. Thus a lower p decreases the risk taking incentive.

For p high enough, the expected payo given a liquidity shock increases with p. As p rises, the bank is further away from its default region, the risk shifting incentive diminishes. The direct e ect becomes negative.

Figure 4 and 5 show the intuition graphically. A low market price p limits the bank's ability to withstand liquidity shocks, driving down its expected payo s regardless of the choice of risk. The e ect of p on the threshold z depends on how p changes the bank payo s with and without risk. The responses of the expected payo s depend on the survival probabilities and the cost of risk taking.

For p su ciently low, the survival probability with risk taking (x_H) is signi cantly higher than with no risk (x_L

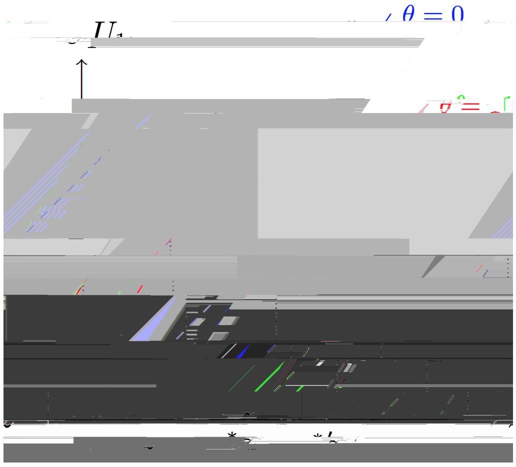

Figure 5: Change in z for a Decrease in p when p is high

enough to cover the repayment to late depositors.

$$^{d}(z;p;R) = \begin{cases} 0 & \text{if } z < \frac{R}{2(1\quad c)} \\ \frac{1}{2}(x_H(z;p;R) + (1\quad)) & \text{if } z \in [\frac{R}{2(1\quad c)}; z(p;R)] \\ x_L(z;p;R) + (1\quad) & o.w.; \end{cases}$$

where x_H and x_L are defined in Equation 43 and 44 above.

For $p > 1$, the market value of bank asset is large enough to withstand liquidity shocks. Banks can always survive date 1. The bank will not default at date 2 as long as z is large enough.

$$^{d}(z;p;R) = \begin{cases} \frac{1}{2}(1\quad x_H(z;p;R)) & \text{if } z < \frac{R}{2(1\quad c)} \\ \frac{1}{2} & \text{if } z \in [\frac{R}{2(1\quad c)}; z(p;R)] \\ 1 & o.w. \end{cases}$$

Given the assumption that depositors will get nothing when bank defaults, the expected return for late depositors when they do not withdraw is $R\ ^{d}(z;p;R)$. They demand deposit rate R such that their expected returns for not withdrawing at date 1 are no less than their outside option of investing in risk-less short term bonds with return r, i.e.,

$$R\ ^{d}(z;p;R)\quad r. \tag{11}$$

In order to convince the depositors not to withdraw early, the bank has to offer a long term return R such that the participation constraint (Equation 11) above hold in equality.

Assumption 2 $(1\quad)(1\quad c)\underline{z} > r.$

to convince a bank not to take risk. As shown perviously, when p declines, the bank can withstand less liquidity shocks. It needs a higher return to compensate its exposure to the liquidity shock. That is , (p) falls as p declines. Unless p is su ciently low, in which case, the bank does not need a high liquidity premium to take risk because its payo in liquidity shocks becomes negligible.

The second term summarizes the \indirect e ect" of secondary market p on z . The channel is through the impact of p on the equilibrium long term deposit rate R .The indirect e ect is always negative. A low p heightens the default probability of the bank and the depositors demand a higher return, R . The higher deposit rate drives down the bank's payo with and without liquidity shock equally. Banks are unambiguously more likely to take high risk. z rises as a result.

The total e ect of secondary market price on the optimal risk taking of the bank is the sum of the two e ects. When p is high enough, both direct and indirect e ects are negative. p encourages banks to take risk unambiguously. Immediately, we have the following result:

PROPOSITION 4 For $p > 1$, both the direct and the indirect e ects are negative, the total e ect

Figure 6: Threshold in the choice of risk $\underline{z}$

depends on which effect is dominant. Figure 6 illustrates separately the two effects of p on z. The left panel depicts the direct effect by fixing $R = 1$. The right panel illustrates the evolution of R given the mean return z. The numerical example suggests that the indirect effect is the driving force of the risk taking behaviors of banks. I formalize the discussion in the following:

Assumption 3 $(1 \quad c)(1 \quad)\underline{z} > \frac{1}{1 \; D(c)}$ where

$$D(c) = 2c \quad (1 \quad c)\left(\frac{1}{2c} \quad \frac{1}{2(1 \quad c)}\right)^2 \quad \left(\frac{1}{2c} \quad 1\right)^2 \quad :$$

PROPOSITION 5 For $p \quad 1$, (i) under Assumption 3, the size of indirect effect always dominates that of the direct effect for $p < \bar{p}$,

$$j\frac{@R=R}{@p=p}j > \frac{@ \; =}{@p=p} > 0;$$

(ii) both indirect and direct effects are negative for $p \; 2 \; [\bar{p}; 1]$; (iii) the total effect

$$\frac{@z(p)=z\,(p)}{@p=p} < 0: \tag{15}$$

for p 1.

uidity. As p falls, the bank entails a higher credit risk. Depositors demand a higher R as a compensation for taking higher risks. The first round of response of R is $\frac{\partial ^d/ ^d}{\partial p/p}$. The higher borrowing cost pushes the bank closer to its default region, inducing more risk taking and driving up the credit risk further. R rises as a second round effect. The size of the second round response is $|\frac{\partial ^d/ ^d}{\partial R/R}|\frac{\partial ^d/ ^d}{\partial p/p}$:

The process goes on until the deposit rate R reaches the market equilibrium. The total impact of p on R :

$$|\frac{\partial R/R}{\partial p/p}| = \frac{\partial ^d/ ^d}{\partial p/p}\ 1 + |\frac{\partial ^d/ ^d}{\partial R/R}| + |\frac{\partial ^d/ ^d}{\partial R/R}|^2 + \ldots : \tag{20}$$

Combining the two cases with different levels of the secondary market price, immediately we have the following result for the optimal risk-taking behavior for banks.

Lemma 3 $z(p; z)$ is decreasing in p and z.

A low secondary market price intensifies the risk shifting incentives and induces the bank to take more risk. A rise in the mean return on the long term asset reduces the credit risk and the deposit rate, leading to a lower threshold z .

3.4 The Aggregate Choice of Risk

Let $\hat{p}(z)$ satisfy that

$$z(\hat{p}; R(\hat{p}; z)) = z: \tag{21}$$

$\hat{p}(z)$ is the minimum secondary market price at which the bank starts to prefer no risk. The optimal choice of risk for individual bank can be rewritten as

$$(p; z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ 0 & \text{if } p \quad \hat{p}(z): \end{cases}$$

The bank chooses risky returns when p is sufficiently low.

Because of the monotonicity of z in p and z, $\hat{p}(z)$ is decreasing in z. The bank has higher incentives to take risk for a low mean return z. Thus a high cutoff price is required in order to convince the bank not to take risk.

Denote n the fraction of banks that take risks in their long term investments. Given p, the aggregate risk-taking behavior of the banking sector is

$$n(p;z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ [0;1] & \text{if } p = \hat{p}(z) \\ 0 & \text{if } p > \hat{p}(z): \end{cases} \tag{22}$$

The banking sector collectively take risk in long term assets when the secondary market price is lower than the cuto price.

3.5 the Market Value for the Illiquid Asset

Buyers are risk averse and they demand a premium for holding asset with risky returns. There is asymmetric information between buyers and sells of assets. Buyers cannot identify the assets with risky returns. Let n^D denote the fraction of banks that take risks. With n^D probability, the buyer expects to obtain an asset with risky returns, which is valued at its expected return with a discount

Figure 7: Market Equilibrium

secondary market is consistent with the initial expectation. In this case, the economy reaches the low volatility equilibrium.

On the contrary, when a high re sale discount in the secondary market price is expected, banks have di culty in paying for their early withdrawals. Close to the default region, banks have more incentives to take risk to boost their payo s. Consequently, the credit risk rises, inducing more banks to take risk. In equilibrium, the whole banking sector takes high risk for their long term investments. With the returns more volatile, the asset price in the secondary market is indeed low. It is the high volatility equilibrium.

Both equilibria are locally stable. Any perturbation to the equilibrium price will not persist. The equilibrium at $p = \hat{p}$ is unstable.

Multiple equilibria exist in this model because of the strategic complementarity gen-

PROPOSITION 5 (Existence of Multiple Equilibria) Given the mean aggregate return z, (i) The low volatility equilibrium exists when $\hat{p}(z) \leq p^D(0; z)$; (ii) The high volatility equilibrium exists when $\hat{p}(z) \geq p^D(1; z)$; (iii) Multiple equilibria exists when (i) and (ii) hold simultaneously.

The low volatility equilibrium exists when the buyers value the assets with risk-less returns high enough to convince the banking sector not to take risk. Meanwhile, the high volatility equilibrium exists when the buyers value the risky assets low enough to encourage collective risk taking of the whole banking sector.

When the social planner takes the payoffs of both depositors and banks into consideration, in the presence of multiple equilibria, the low volatility equilibrium is more efficient than the high volatility equilibrium. The reason is that while the depositors expected payoff are equal in both equilibria[24], the bank's payoff is always higher in the low volatility equilibrium.[25]

3.7 Comparative Statics

This subsection studies how the aggregate productivity z, the probability of a liquidity shock and the cost of taking risk c affect equilibrium outcomes.

3.7.1 Aggregate Productivity z

Aggregate productivity boosts long term payoffs of banks and weakens their incentive to take risk. In the presence of a high z, a large fire sale discount in the secondary market price is needed to convince the banking sector to take risk. As shown previously, $\hat{p}(z)$ is decreasing in z.

Let z_G denote the threshold productivity which satisfies that

$$\hat{p}(z) = p^D(0; z): \tag{23}$$

Because $\hat{p}(z)$ is decreasing in z, the condition for the existence of the good equilibrium (with low volatility) is equivalent to $z > z_G$.

Similarly, let z_B denote the threshold productivity such that

$$\hat{p}(z) = p^D(1; z): \tag{24}$$

[24] This is followed by the depositors participation constraint.

[25] It can be shown that

$$U_0(z; 1)|_{p=p^D(1;z)} \leq U_0(z; 1)|_{p=\hat{p}(z)} = U_0(z; 0)|_{p=\hat{p}(z)} \leq U_0(z; 0)|_{p=p^D(0;z)}:$$

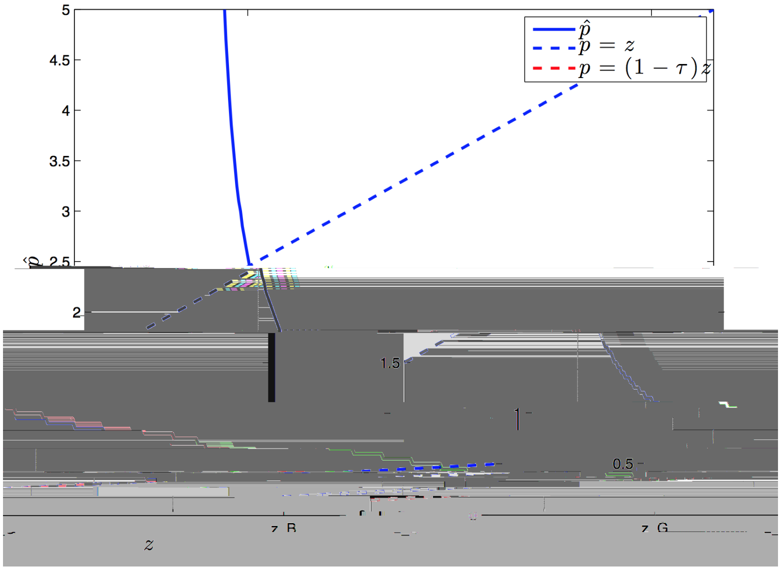

Figure 8: Threshold $\hat{p}$ as a function of z

The condition for the bad equilibrium (with high volatility) is equivalent to $z < z_B$.

Note that $z_G < z_B$. Multiple equilibria exists when $z \in [z_G; z_B]$. In this range, expectations can be self-fulfilling. See Figure 8.

The effect of aggregate productivity on risk taking decision by banks works through two channels. Given R , a low mean return z encourages banks to take advantage of the limited liability to boost payoffs. More banks take risk. Expecting this, depositors demand higher rate R to compensate for the default risk, which intensifies the risk-shifting incentive further. $\hat{p}$ rises as z falls. A wider range of secondary market price is admissible for the systemic risk-taking behavior of the banking sector. The high volatility equilibrium becomes more likely. When the mean return z is sufficiently low, the economy can potentially be trapped in the inefficient equilibrium with severe fire sale discounts and high volatility in asset returns. The Figure 8 illustrates the domain of z for multiple equilibria in a numerical example with given c and .

3.7.2 Changes in c

c denotes the cost of taking high risk, i.e., c p

Figure 9: Threshold $\hat{p}$ with Changes in c

As c rises, a lower secondary market price is needed to convince the whole banking sector to take risk. The left panel in Figure 9 depicts the cuto price $\hat{p}$ with a change in c. As c rises, both boundary conditions for the good and bad equilibria (z_G and z_B) decreases. With a given , the right panel illustrates the two boundary conditions z_G and z_B in c. In between is the region for multiple equilibria. Because c reduces risk-taking incentive, as c rises, the good equilibrium becomes more likely and the bad equilibrium becomes less likely.

3.7.3 Changes in

is the probability that a liquidity shock hits the banking sector at date 1. The e ect of on the expected payo s of banks is twofold. On the one hand, a high depresses the bank payo s because with the rising needs for early withdrawals, the banks are more likely to sell assets for liquidity. On the other hand, a high improves the bank payo s because as more depositors are expected to withdraw early, less repayment is needed in the long term.

The total e ect of on the risk decisions of banks depends on the secondary market price. For $p > 1$, the bank can always withstand liquidity shocks at date 1. The second

Figure 10: Threshold $\hat{p}$ with Changes in

e ect dominates. The expected payo s increase in because banks expect lower total repayments to the late depositors at date 2. Therefore a high discourages risk-taking behavior. As banks expect a higher probability of liquidity shocks, they would not take risk unless the secondary market price is low enough, i.e. $\hat{p}$ falls.

For p 1, with an less liquid secondary market, banks become more concerned over the forced re sales. The rst e ect dominates. A high lowers bank payo s and encourages risk taking. Meanwhile, since defaults become more likely, depositors demand a higher R . Banks are pushed even closer to their default region, intensifying their incentives to take risk. $\hat{p}$ increases with . A wider range of secondary market prices is admissible for the existence of the high volatility equilibrium.

The left panel in Figure 10 illustrates the cuto price $\hat{p}$ in z. a ects $\hat{p}$ di erently depending on the secondary market price. As discussed above, an increase in reduces risk-taking when p is high. $\hat{p}$ decreases, rendering the good equilibrium more likely. On the contrary, it encourages risk-taking when p is low. $\hat{p}$ increases, raising the likelihood of the bad equilibrium. The right panel illustrates the boundary conditions for the good and bad equilibria. An increase in the liquidity shock probability raises the likelihood for both equilibria, leading to an expansion of the region for multiple equilibria.

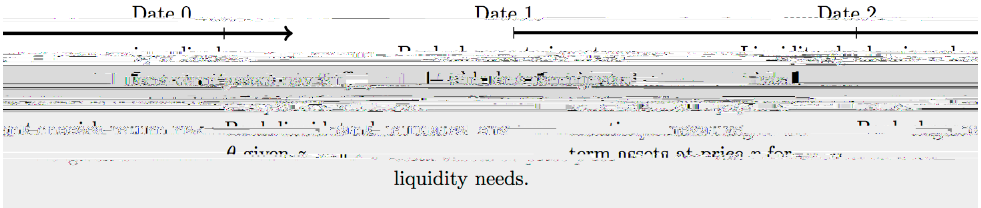

Figure 11: Time-line with Ex Ante Choice of Liquidity

4 The Model with Ex Ante Liquidity

This section studies an extended version of the model by incorporating an ex ante choice of liquid asset holding. That means, at date 0, the bank can invest in a safe and perfectly liquid asset, which generates return 1 from date 0 to date 1. Now, banks are making two decisions at date 0: given mean return z, banks choose both the quantity and the risk of their long term investments.[26]

With the additional choice of liquidity holding, the time line in the extended model is illustrated in Figure 11.

When the bank is subject to liquidity shocks, it is natural to discuss bank's problem where the bank has the option to hold liquid assets ex ante as a precautionary bu er. Incorporating the ex ante choice of liquidity also provides more room for the discussion of some maco-prudential policies.

The e ect of liquidity holding on the choice of risk is twofold. On one hand, the holding of liquidity prevents long term assets from liquidating so it boosts long term payo s of banks, which lower their risk-shifting incentives. On the other hand, when the bank hoards liquidity in stead of engaging in long term investments, the bank payo is negatively a ected. Consequently, the bank tends to take more risk. Which e ect dominates will depend on the relative strengths of the cost and the bene t of holding liquidity. The model suggests that the total e ect of liquid asset holding on the risk taking by banks is non-monotonic. It will depend on the secondary market price.

[26] The choice of liquid asset holding can happen before the mean return z. In this case, assuming that z is drawn from a distribution $F(z)$ on $[\underline{z}; z]$. This setting means that the bank commits certain funds to some long term projects and then chooses the risk associated with the project given the realization of z. The timing here does not alter the results signi cantly.

4.1 Bank's Problem

Now banks have two decisions to make. Given an aggregate mean return z, banks decide how much liquidity to hold and the riskiness on their long term assets simultaneously:

$$\max_{l;\,\sigma\in[0;1]} U_0(l; z; \sigma). \tag{25}$$

U_0 is the expected payoff of the bank,

$$U_0(l; z; \sigma) = E_x E_s \max\{y(l; z; x; s; \sigma); 0\}; \tag{26}$$

where the ex post payoff

$$y(l; z; x; s; \sigma) = \begin{cases} (1-l)(1+s\sigma)(1-\delta(\sigma))z + (l-x)r - (1-x)\bar{R} & \text{if } x < l \\ (1-l-\frac{x-l}{p})(1+s\sigma)(1-\delta(\sigma))z - (1-x)\bar{R} & \text{o.w.} \end{cases}$$

In order to fulfill the liquidity needs, the bank liquidates its long term asset after it exhausts its liquidity buffer l.

4.1.1 Choice of Risk

I first solve for the choice of risk by banks for given liquidity holding l and then solve for the optimal liquidity holding. Similar to the previous analysis, the optimal choice of risk would take corner solutions, i.e., $\sigma \in \{0; 1\}$.

PROPOSITION 6 (Choice of Risk) Given mean return z and the choice of liquidity l, bank chooses riskiness of its long term assets according to

$$\sigma(l; z; p; \bar{R}) = \begin{cases} 1 & \text{if } z < z^*(l; p; \bar{R}) \\ 0 & \text{if } z \geq z^*(l; p; \bar{R}) \end{cases} \tag{27}$$

where $z^*(l; p; \bar{R})$ is the threshold mean return that equates the expected payoffs with and without risk,

$$U_0(l; z; 1) = U_0(l; z; 0). \tag{28}$$

Because the expression for $z^*(l; p; \bar{R})$ is complicated to compute, there is no closed-form solution for z^*. However, we can discuss the evolution of z^* in its arguments.

Lemma 4 $z^*(l; p; \bar{R})$ is increasing in l for small enough δ.

$\frac{1}{p}$ long term assets from liquidation. On the other hand, liquidity holding incur a cost since the bank has to give up returns from long term assets as long as it does not default. In a liquid secondary market with $p > 1$, the cost of liquidity holding always exceeds the benefit for all realizations of z. It is optimal not to hold any liquidity ex ante.

In an illiquid secondary market with p 1, holding additional liquidity saves more than one unit ($\frac{1}{p}$ 1) of assets from liquidation. The gain from holding a stronger liquidity buffer can potentially outweigh the cost. Moreover, as p falls, it becomes increasingly difficult to sell long term assets for liquidity. Banks benefit more from holding liquid assets ex ante. Banks prefer holding non-zero liquidity when p is sufficiently low.

For sufficiently small, the ex ante illiquidity risk is small. That is, the bank is very likely to survive date 1. In this case, the bank has no incentive to hold liquid assets ex ante even for a low p. Therefore, the extended version of the problem coincides with the basic model.

4.2 The Determination of R

Same as in the basic model, depositors observe the realized mean return z and the asset price in the secondary market p. They cannot observe the decisions (on liquid assets holding or risk taking) made by their individual banks. Nor can they observe the size of the liquidity shock x. They form the probability of no default, taking into account the optimal choices of risk and liquidity holding by banks. The probability of no default from depositor's perspective ^{d}:

$$^{d}(l \; ; z; p; R \;) = \begin{cases} Pr(y(l \end{cases}$$

encourages risk-taking through a combination of direct and indirect e ects.

4.3 The Aggregate Risk Taking of the Banking Sector

Due to the monotonicity of z in p, similar to the analysis in the basic model, there exists a threshold secondary market price $\hat{p}(z)$ and it satis es that

$$z = z\ (0; p; R\ (p; z)): \tag{32}$$

The banking sector take high risk in long term investments when p is su ciently low, i.e., $p < \hat{p}(z)$.

Given secondary market price p and the mean return z, the fraction of banks that take risk in equilibrium is

$$n(p; z) = \begin{cases} 1 & \text{if } p < \hat{p}(z) \\ [0; 1] & \text{if } p = \hat{p}(z) \\ 0 & \text{if } p > \hat{p}(z): \end{cases} \tag{33}$$

4.4 Characterization of the Equilibrium

The buyers' valuation of the long term asset is in Equation 5.

De nition of the equilibrium: The rational expectation equilibrium is de ned by (l ; ; n ; R ; p) such that

a) l and are the optimal choices of liquidity and risk in asset returns chosen by the banks given p and R .

b) R

In the low volatility equilibrium, the secondary market is liquid with $p > 1$. Expecting low volatility in asset returns, banks will not hoard liquid asset ex ante because they expect that it will be easy to satisfy the liquidity need of its depositors. The equilibrium behaves exactly the same as the one in the basic model. With all its funds tying to the long term investments, the bank has less incentive to take risk because the cost of risk taking exceeds the benefit. So the volatility of asset returns is low.

High volatility equilibrium exists when the buyers valuation for the risky assets is low enough to induce the banking sector to take risk. Risky asset returns is associated with lower price in the secondary market. With low , the likelihood of a positive liquidity shock is low, and banks would not exploit the extra option of storing values. In response to a low price, banks choose risky projects. As in the basic model, the expectation of a low price in the secondary market can be self-fulfilling.

4.5 Liquidity Requirement

Because the equilibrium with the self-fulfilling crisis generates welfare loss, this section analyzes how some standard macro-prudential policies affect the financial market efficiency in the context of the model. Specifically, consider a liquidity requirement imposed on the whole banking sector. Banks are required to hold at least units of liquid assets for each unit of deposits. So there is an additional constraint in the bank's problem in Equation 25,

$$l \quad :$$

For sufficiently low , banks have no incentives to hold liquid assets ex ante. The liquidity requirement always has binding power.

banking sector rises, depositors expect a fall in the credit risk as well as in the long term deposit rate. Combining the two effects, in the presence of the liquidity requirement, the threshold in risk-taking behavior z falls for p 1 and rises for $p > 1$.

When $p > 1$, all banks have enough liquidity to cover the early withdrawals and they can always survive date 1. Banks only default when they take risky investment and experience the negative productivity shock at date 2, which takes place with $1/2$ probability. Additional liquidity holding will affect neither the credit risk nor the deposit rate. The effect of liquidity regulation on z is identical to the effect of individual liquidity holding as discussed in the previous section. When secondary market price is expected to be high, a liquidity requirement will be counterproductive by encouraging the banking sector to take more risk.

When p 1, the liquidity requirement > 0 can discourage risk taking by banks. The intuition in the following. In addition to positive effect of liquidity holding on risk-taking at individual levels as discussed previously, the increase in the aggregate liquidity holding affects the equilibrium deposit rate R. With $p < 1$, early withdrawals may potentially exceed the market value of bank asset ($p(1 \quad l) + l < 1$), in which case the bank will be forced to default at date 1 due to a liquidity shortage. Holding more liquid assets strengthens bank's ability to withstand liquidity shocks and reduces the illiquidity risk and hence the credit risk of the bank. Therefore, bank payoffs improve as depositors demand a lower deposit rate. In this case, with improvement in bank payoffs, a liquidity requirement can rein in risk taking by banks.

4.5.1 Effect of Liquidity Regulation on Multiple Equilibria

Given liquidity regulation , the aggregate choice of risk of the banking sector becomes

$$n(p; z) = \begin{cases} 1 & \text{if } p < \hat{p}(z; \ ) \\ [0; 1] & \text{if } p = \hat{p}(z; \ ) \\ 0 & \text{if } p > \hat{p}(z; \ ) \end{cases} \tag{36}$$

where the cutoff price $\hat{p}(z; \ )$ is defined as

$$z = z (\ ; p; R \ ): \tag{37}$$

Proposition 8 (The Effect of on Aggregate Risk Taking) *For small enough, the effect of liquidity requirement on the aggregate risk taking:*

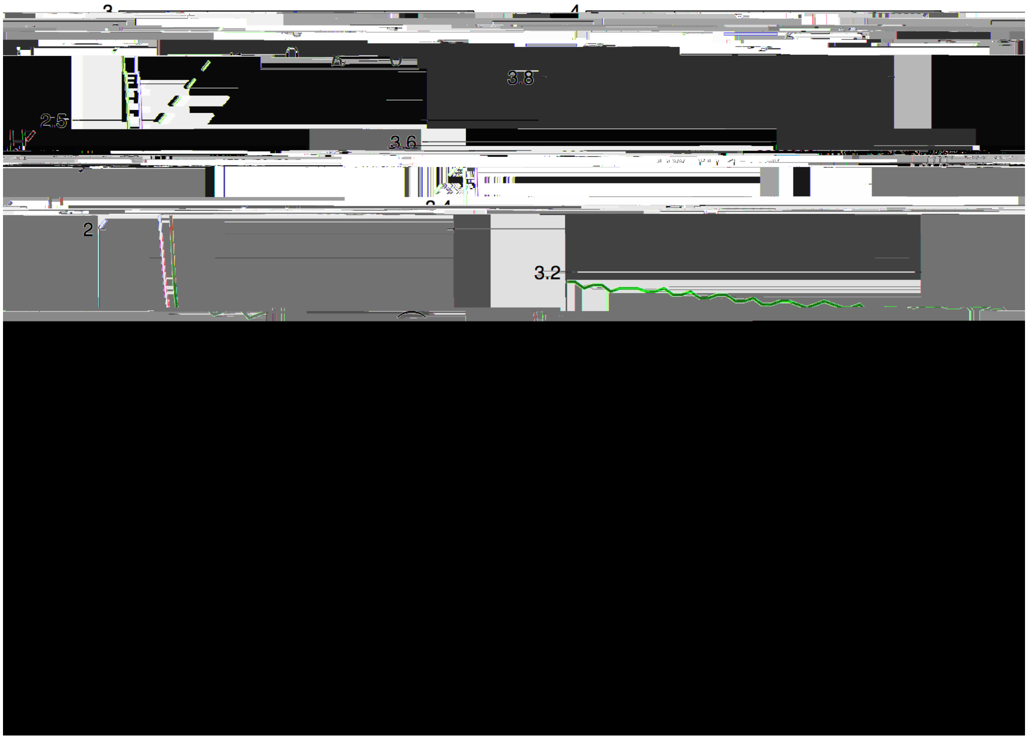

Figure 12: The Effect of on risk taking and on equilibrium

(i) For p > 1,

$$\frac{\partial \phi(z;\ )}{\partial} > 0. \tag{38}$$

(ii) For p 1,

$$\frac{\partial \phi(z;\ )}{\partial} < 0. \tag{39}$$

The proposition can be shown immediately by Lemma 6. The model suggests that the effect of a liquidity requirement is ambiguous in improving financial stability, as shown in Figure 12.

When the secondary market price is low, banks are holding liquid assets at a level which is lower than the social optimal level. The reason is that banks do not internalize the effect of their liquidity holdings on the equilibrium credit risk and deposit rate. Liquidity requirement reduces the risk-shifting incentives by reducing the credit risk and hence the borrowing cost of banks and improving bank payoffs. With less willingness to take risk, existence of the high volatility equilibrium, as shown in the right panel of Figure 12. Liquidity requirement is welfare-improving by enhancing nancial stability for p 1.

However, when the secondary market price is expected to be high, the liquidity requirement is potentially counterproductive in stabilizing the nancial system. The cost of forfeited long term returns outweighs the bene t from a stronger liquidity bu er. Liquidity requirement restrains banks from making long term investments and dampen their long term payo s. As a result, banks are incentivized to rely on risky returns to boost payo s. The excessive risk taking reduces the occurrence of the low volatility equilibrium. An increase in would intensify the risk-shifting incentives further and render the low volatility equilibrium less likely to exist. In Figure 12, the cuto price shifts up for $p > 1$ and less z's are admissible for the existence of the low volatility equilibrium.

In sum, the liquidity requirement reduces the occurrence of both the good and bad equilibria. The model suggests that the liquidity requirement imposes a tradeo in the policy making process: improving nancial stability in bad times (by reducing the high volatility equilibrium) vs encouraging excess risk taking in good times (by reducing the high volatility equilibrium).

4.5.2 Discussions on the Counter-cyclical Liquidity Requirement

The recent nancial crisis sheds light on the pro-cyclicality of behaviors in the nancial market. There has been a growing consensus on the implementation of the counter-cyclical regulations in promoting the resilience of the nancial system.

The model provides theoretical arguments in favor of the implementation of counter-cyclical liquidity requirement, in which the liquidity requirement is raised during economic upturns and lowered during economic downturns. As discussed above, a constant liquidity requirement imposes a tradeo between improving nancial stability (by reducing the high volatility equilibrium) vs. encouraging excess risk taking (by reducing the low volatility equilibrium). The counter-cyclical liquidity requirement is shown to promote nancial stability by improving the tradeo . The intuition is the following.

The relative strength of the two sides of the tradeo of a liquidity requirement di ers according to di erent levels of aggregate productivity. When the aggregate productivity is high, a high liquidity requirement guarantees that banks hold enough liquid assets as precautionary bu ers for the upcoming liquidity shocks. The requirement could e ectively rein in excessive risk taking when re sales are expected. Meanwhile, it would not encourage risk taking signi cantly because during economic booms, long term projects generate high returns and the bank total payo s are less a ected by holding more liquid assets.

On the contrary, when the aggregate productivity is low, a low liquidity requirement would reduce the incentive distortion and discouraging excessive risk taking when the secondary market price is high. Meanwhile, the requirement may not further intensify the risk taking incentives because during economic downturns, bank payoffs are already low.

The model suggests that the liquidity requirement should be made counter-cyclical. In this way, we can utilize the beneficial effect of a liquidity requirement while minimize its counter-productive effect on the financial market in order to promote financial stability.

In fact, the optimal liquidity requirement implied by the model is given by solving the constrained first best problem that maximizes the aggregate expected payoff of banks,

$$\max U_0(\; ; z \; ; \quad): \tag{40}$$

subject to the depositors participation constraint,

$$R \quad ^{d}(\; ; z; p; R \;) = r \tag{41}$$

and the optimal risk taking of individual banks,

$$(\; ; z; p; R \;) = \begin{cases} 1 & \text{if } z < z \; (\; ; p; R \;) \end{cases}$$

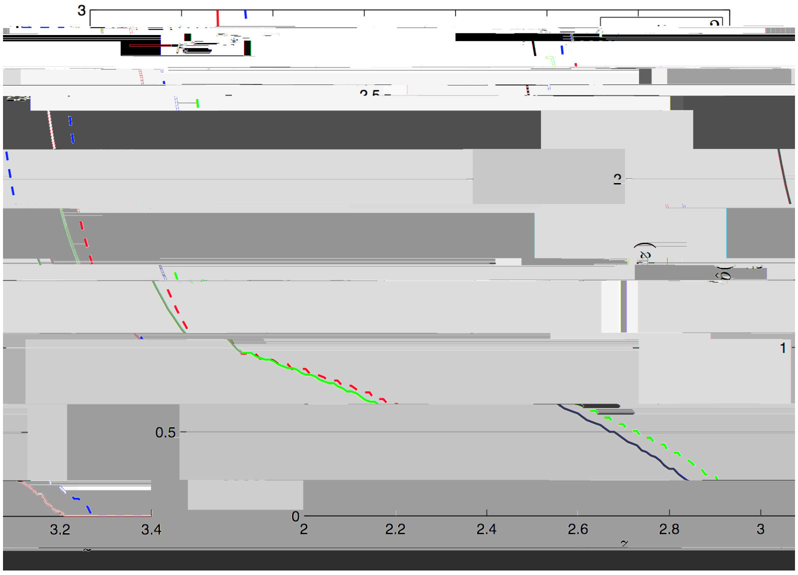

Figure 13: The E ect of Optimal Liquidity Requirement on risk taking

other short term creditors, because their returns from outside options are limited by the zero-interest cash. Due to the downward rigidity in the returns of creditors from outside options, the bank pro ts are squeezed. With lower pro t margins, the bank would prefer investing in riskier projects for the bene t of risk-shifting to boost its payo . As a result, the credit risk of banks rises. In stead of a lower borrowing cost intended by the sub-zero rate policy, the banking sector could potentially face a higher borrowing cost, leading to further disruptions in the banking sector. The incentive to \search for yield" [28] imposes a challenge for the stability of the nancial system.

References

[1] Acharya, V., 2009. \ A theory of systemic risk and design of prudential bank regulation". Journal of Financial Stability 5, 224-255.

[2] Acharya, Viral, Douglas Gale, and Tanju Yorulmazer, 2011. \ Rollover risk and market freezes," Journal of Finance 66, 1177?1209.

[3] Allen, Franklin, and Elena, Carletti, 2008.\The role of liquidity in nancial crises," 2008 Jackson Hole Conference Proceedings, Federal Reserve Bank of Kansas City

[4] Bachmann, R., and C. Bayer, 2013. \ Wait-and-See" business cycle?," Journal of Monetary Economics, 60(6), 704 - 719.

[5] Bachmann, R., S. Elstner, and E. R. Sims, 2013. \ Uncertainty and Economic Activity: Evidence from Business Survey Data," American Economic Journal: Macroeconomics, 5(2), 217-249.

[6] Bachmann, R., and G. Moscarini, 2012. \ Business Cycles and Endogenous Uncertainty," Working paper.

[7] Bernanke, Ben S., and Alan Blinder. 1988. \Credit, Money, and Aggregate Demand.? American Economic Review, 78(2): 435?39.

[8] Bigio, Saki, 2015. \Endogenous liquidity and the business cycle," American Economic Review, 105, 1883-1927.

[9] Bloom, N., 2009. \The Impact of Uncertainty Shocks," Econometrica, 77(3), 623-685.

[10] Bloom, N., M. Floetotto, I. Saporta-Eksten, and S. J. Terry, 2012. \ Really Uncertain Business Cycles," Working paper, Stanford University and Duke University.

[11] Brunnermeier, Markus K., 2009. \Deciphering the Liquidity and Credit Crunch 2007?2008," Journal of Economic Perspective, 23, 77-100.

[12] Brunnermeier, Markus K., and Lasse Heje Pedersen. 2009. \Market Liquidity and Funding Liquidity.," Review of Financial Studies, 22, no.6, 2201?38.

[13] Dell'Ariccia G., L. Laeven, and G. Suarez, 2016. \Bank Leverage and Monetary Policy?s Risk-Taking Channel: Evidence from the United States?, Journal of Finance, forthcoming.

[14] Diamond, Douglas W., and Philip H. Dybvig, 1983. \Bank runs, deposit insurance, and liquidity," Journal of Political Economy 91, 401-419.

[15]

[30] Ioannidou V., S. Ongena, and J.-L. Peydr09. \ Monetary policy and subprime lending: a tall tale of low federal funds rates, hazardous loans and reduced loan spreads?, European Banking Centre Discussion PaperNo. 45, 2009.

[31] Jimz G., S. Ongena, J.-L. Peydr J. Saurina, 2014. \Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the E ects of Monetary Policy on Credit Risk-Taking??, Econometrica, Vol. 82, No. 2, pp. 463-505.

[32] Jensen, M.C. and Meckling, W.H., 1976. \Theory of the rm: Managerial behavior, agency costs and ownership structure" Journal of Financial Economics 3, 305-360.

[33] Krishnamurthy, Arvind, 2010. \How debt markets have malfunctioned in the crisis," Journal of Economic Perspectives 24, 3?28.

[34] Kurlat, Pablo, 2013. \Lemons Markets and the Transmission of Aggregate Shocks." American Economic Review 103, 1463-89.

[35] Landier, Augustin, David Sraer and David Thesmar, 2011. \ The risk-shifting hypothesis: Evidence from sub-prime originations." 12th Jacques Polak Annual Research Conference

[36] Maddaloni A. and J.-L. Peydr14. \Bank Risk-Taking, Securitization, Supervision and Low Interest Rates: Evidence from the Euro Area and U.S. Lending Standards?, Review of Financial Studies, Vol. 24, No. 6, pp. 2121-165.

[37] Malherbe, Frederic, 2014. \Self-Ful lling Liquidity Dry-Ups." Journal of Finance 69, 1443-1508.

[38] Martinez-Miera, David, and Rafael Repullo, 2015. \Search for Yield ." Working Paper

[39] Morris, Stephen, and Hyun Shin, 2004. \Liquidity black holes." Review of Finance 8, 1-18.

[40] Morris, Stephen, and Hyun Shin, 2010. \Illiquidity Component of Credit Risk." Working Paper.

[41] Navarro, Gaston, 2014. \Financial Crises and Endogenous Volatility." Working Paper

[42] Orlik, A., and L. Veldkamp, 2014. \Understanding Uncertainty Shocks and the Role of Black Swans," Working paper.

[43] Rajan R., \Has nancial development made the world riskier??, European Financial Management, 12(4), 2006, pp. 499?533.

[44] Repullo, Rafael, 2005. \Liquidity, risk taking and the lender of last resort." International Journal of Central Banking 1, 47-79.

[45] Shleifer, Andrei, and Vishny, Robert W. 1992. \Liquidation Values and Debt Capacity: A Market Equilibrium Approach.? Journal of Finance, 47(4): 1343?66.

[46] Shleifer, Andrei, and Vishny, Robert W. 2011. \Fire Sales in Finance and Macroeconomics.? Journal of Economic Perspectives, 25, 1, pp 29?48.

[47] Stiglitz, J., Weiss, A., 1981. \Credit rationing in markets with imperfect information". American Economic Review 71, 393?410.

[48] Summers, Lawrence H., 2014. \U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound". Business Economics 49, 65?73wrence H., Rdk528(L.,-374(2

Appendix

Proof of corner solutions With Assumption 1, $U_0(z; \ )$ reaches its maximum at the corners of will not default as long as z R .

$$\frac{@U_0(z; \ )}{@} = \ cz \int_0^1 (1 \ \frac{x}{p})dx + (1 \ ) < 0:$$

For p 1, x(z; s;) < 1 for s = 1 and 1.

$$\frac{@U_0(z; \ )}{@} = \ cz \int_0^{x(z; \ 1; \ )} (1$$

(ii) For $p > 1$, for $z > R$

$$U_0(R\ ; z; 1) = (1 \quad c) \quad z \quad \frac{R}{2(1 \quad c)} \quad \frac{}{2} + (1 \quad) \quad + (1 \quad c)\frac{}{2}(1 \quad \frac{1}{p})z$$

and

$$U_0(R\ ; z; 0) = (z \quad R\ ) \quad \frac{}{2} + (1 \quad) \quad + \frac{}{2}(1 \quad \frac{1}{p})z:$$

Otherwise, for $z \quad R$, $U_0(z; 1) = U_0(z; 0) = 0$.

Solving for z (i) For $p \quad 1$, for $z > R$, z satis es

$$(1 \quad c) \quad z \quad \frac{R}{2(1 \quad c)} \quad \frac{}{2} x_H(p; R\ ; z) + (1 \quad) \quad = (z \quad R\ ) \quad \frac{}{2} x_L(p; R\ ; z) + (1 \quad) \quad :$$

Dividing both sides by R ,

$$(1 \quad c) \quad \frac{1}{2(1 \quad c)} \quad \frac{}{2} x_H(p; 1; \ ) + (1 \quad) \quad = (\quad 1) \quad \frac{}{2} x_L(p; 1; \ ) + (1 \quad) \quad :$$

The equation gives unique solution for . $z = (p)R$.

Proof of the uniqueness of z There are two steps to prove the existence of a unique z .

Step 1: $z = R \quad \frac{1}{2c}$.

Note that (p) is rst increasing then decreasing in p for $p < 1$[31], $(p) \quad \min f \ (0); \ (1) g = \frac{1}{2c}$, then $z \quad \frac{1}{2c} R$.

Step 2: for $z \quad z \quad R = (2c)$, it can be shown that

$$\frac{@U_0(z; 1)}{@z} < \frac{@U_0(z; 0)}{@z}:$$

Note that

$$\frac{@U_0(z; 1)}{@z} = (1 \quad c) \quad \frac{}{2} x_H(p; R\ ; z) + (1 \quad) \quad + (1 \quad c) \quad z \quad \frac{R}{2(1 \quad c)} \quad \frac{}{2} \frac{@x_H(p; R\ ; z)}{@z}:$$

and

$$\frac{@U_0(z; 0)}{@z} = \quad \frac{}{2} x_L(p; R\ ; z) + (1 \quad) \quad + (z \quad R\ ) \frac{}{2} \frac{@x_L(p; R\ ; z)}{@z}:$$

To show $\frac{@U_0(z;1)}{}$

(ii) For $p > 1$, z satis es

$$(1 \quad c) \quad z \quad \frac{R}{2(1 \quad c)} \quad \frac{}{2} + (1 \quad) \quad + (1 \quad c)\frac{}{2}(1 \quad \frac{1}{p})z = (z \quad R\ ) \quad \frac{}{2} + (1 \quad) \quad + \frac{}{2}(1 \quad \frac{1}{p})z:$$

Dividing both sides by R ,

$$(1 \quad c) \quad \frac{1}{2(1 \quad c)} \quad \frac{}{2} + (1 \quad) \quad + (1 \quad c)\frac{}{2}(1 \quad \frac{1}{p}) \quad = (\quad 1) \quad \frac{}{2} + (1 \quad) \quad + \frac{}{2}(1 \quad \frac{1}{p}) :$$

solving yields,

$$= \frac{1}{2c}\frac{\frac{1}{2} \quad + 1}{(1 \quad \frac{1}{2p}) \quad + 1}:$$

Then, $z \ = \ (p)R$.

Proof of Proposition 2: (i) For $p > 1$ it is trivial to prove that (p) is decreasing in p.

(ii)For p 1, given R = 1, z = (p). (

(1 c(

1

z

2

c(

The numerator is a quadratic and concave function in z. For $z > R$, there exists a unique $\hat{z}(p)$ that makes the numerator equal to zero. $\hat{z}(p)$ is decreasing in p. At $p = 0$, $z(0; R) < \hat{z}(p)$ or equivalently $\sqrt{1 \quad cx_H} \quad x_L > 0$. So $z(p; R)$ is increasing. Until p reaches $\bar{p}$ that makes $z(p; R) = \hat{z}(p)$, or equivalently $\sqrt{1 \quad cx_H} \quad x_L = 0$. After that, for $p > \bar{p}$, $z(p; R) > \hat{z}(p)$, or $\sqrt{1 \quad cx_H} \quad x_L < 0$. That is, $z(p; R)$ decreases.

Proof of PROPOSITION 3 (The Existence of R) a) At $R = r$, R is, With the assumption 3, $(1 \quad c)(1 \quad)\underline{z} > \frac{1}{1 \quad D(c)}$, the inequality

$$\frac{@}{@p}\frac{1}{\quad} < \frac{@^d}{@p}\frac{1}{\quad^d}$$

holds for all p.

Proof of Proposition 6 The expected payo s in liquidity shocks with and without risk are expressed below respectively,

$$
\begin{aligned}
U_0^L(l; R \ ; z; 1) &= \frac{1}{2}\int_0^{\ (p)} \max \ y_H(l; R \ ; z; x; 1); 0 \ dx \\
&= \frac{1}{2}\int_0^{l} (1 \quad l)2(1 \quad c)z + (l \quad x)r \quad (1 \quad x)R \\
&\quad + \frac{1}{2}\int_l^{x_H} (1 \quad l \quad \frac{x \quad l}{p})2(1 \quad c)z \quad (1 \quad x)R \\
&= (1 \quad l)(1 \quad c)zx_H + \frac{l^2}{4}r \quad \frac{(1 \quad c)z}{2p}(x_H \quad l)^2 \quad (2 \quad x_H)x_H\frac{R}{4}
\end{aligned}
\tag{47}
$$

and

$$
\begin{aligned}
U_0^L(l; R \ ; z; 0) &= \int_0^{\ (p)} \max \ y_H(l; R \ ; z; x; 0); 0 \ dx \\
&= \int_0^{l} (1 \quad l)z + (l \quad x)r \quad (1 \quad x)R + \int_l^{x_L} (1 \quad l \quad \frac{x \quad l}{p})z \quad (1 \quad x)R \\
&= (1 \quad l)zx_L + \frac{l^2}{2}r \quad \frac{z}{2p}(x_L \quad l)^2 \quad (2 \quad x_L)x_L\frac{R}{2}
\end{aligned}
\tag{48}
$$

Note that $x^H(l; p; R \ ; z)$ and $x^L(l; p; R \ ; z)$ are de ned respectively as

$$x^H(l; p; R \ ; z) = \min f\frac{\frac{(p;l)}{p}z \quad \frac{R}{2c}}{\frac{1}{p}z \quad \frac{R}{2c}}; 1g \tag{49}$$

and

$$x^L(l; p; R \ ; z) = \min f\frac{\frac{(p;l)}{p}z \quad R}{\frac{1}{p}z \quad R}; 1g: \tag{50}$$

When the bank is not hit by liquidity shocks, the expected payo s with and without risk,

$$
\begin{aligned}
U_0^{NL}(l; R \ ; z; 1) &= \frac{1}{2}y_H(l; R \ ; z; 0; 1) \\
&= (1 \quad c) \quad (1 \quad l)z \quad \frac{R}{2(1 \quad c)} \quad + \frac{1}{2}lr;
\end{aligned}
\tag{51}
$$

and

$$U_0^{NL}(l; R\ ; z; 0) = \frac{1}{2} y_H (l; R\ ; z; 0; 0) \qquad (52)$$

$$= (1 \quad l) z \quad R \ + lr:$$

The expected payo s are given by

$$U_0(l; R\ ; z;\ ) = \ U_0^L(l; R\ ; z;\ ) + (1 \quad) U_0^{NL}(l; R\ ; z;\ ) \qquad (53)$$

where $\ 2 f 0; 1g$.

The threshold mean return is given by $U_0(l; R\ ; z; 0) = U_0(l; R\ ; z; 1)$.

(i) For $p > 1$, it is easy to show that

$$z\ (l; p; R\ ) = \frac{1}{2c} \frac{1}{1 \quad l} \frac{R \quad \frac{l^2}{2} r \quad (1 \quad) lr}{1 \quad \frac{}{2p}(1 \quad l)}:$$

The uniqueness of z is easy to show because $\frac{@U_0(l;R\ ;z;1)}{@z} < \frac{@U_0(l;R\ ;z;0)}{@z}$ for all $z > \frac{R \quad lr}{1 \quad l}$.

(ii) For $p \quad 1$,

$$\frac{@U_0(l; R\ ; z; 1)}{@z} \quad \frac{@U_0(l; R\ ; z; 0)}{@z} = (1 \quad l)(\ l\ + 1 \quad)(\ c)$$

$$+ \quad (1 \quad l)^2 \quad (1 \quad c) x_0^H \quad x_0^L \quad \frac{1}{2p} \quad (1 \quad c) x_0^{H\,2} \quad x_0^{L\,2}$$

$$< 0$$

where x_0^H and x_0^L are de ned in the basic model. The inequality holds for small enough[32]. Then, there is a unique z that solves the date 1 problem of the bank.

Proof of Lel6f 13392r40 Td [(z)]TJ/F59 7.97 9.10584627 11.9552 Tf1a5243.062 0 Td [( )]TJ/F55

z

For p 1. Taking derivative of U_1 with respect to l,

$$\frac{@U_0(l;R\ ;z\ ;1)}{@l} = \ x^{H}\ + (1\quad)\ (\ (1\quad c)z\ ) + (\ l\ + (1\quad))\frac{r}{2} + \frac{1\quad c}{p}z\ (1\quad l)x_0^H$$

$$\frac{@U_0(l;R\ ;z\ ;0)}{@l} = \ x^{L}\ + (1\quad)\ (\ z\ ) + (\ l\ + (1\quad))\,r + \frac{z}{p}(1\quad l)x_0^L\ :$$

Then the di erence

$$\frac{@U_0(l;R\ ;z\ ;1)}{@l}\quad\frac{@U_0(l;R\ ;z\ ;0)}{@l} = (\ l\ + (1\quad))\,(cz\quad\frac{1}{2}r) + (\frac{1}{p}\quad 1)(1\quad l)\ z\quad(1\quad c)x_0^H\quad x_0^L$$

It is positive when

$$< \frac{cz\quad\frac{1}{2}r}{(1\quad l)\ \ (cz\quad\frac{1}{2}r) + (\frac{1}{p}\quad 1)z\ (x_0^L\quad(1\quad c)x_0^H\ )} =$$

Because $x_0^L\quad(1\quad c)x_0^H < cp$, for

$$< \frac{cz\quad\frac{1}{2}r}{(1\quad l)\ \ (cz\quad\frac{1}{2}r) + (1\quad p)cz} < \quad ;$$

we have

$$\frac{@z}{@l} > 0:$$

For p > 1. Taking derivative of U_1 with respect to l,

$$\frac{@U_0(l;R\ ;z\ ;1)}{@l} = \ (1\quad c)z\ + (\ l\ + (1\quad))\frac{r}{2} + \frac{1\quad c}{p}z\ (1\quad l)$$

$$\frac{@U_0(l;R\ ;z\ ;1)}{@l} = \ z\ + (\ l\ + (1\quad))\,r + \frac{z}{p}(1\quad l):$$

Then the di erence

$$\frac{@U_0(l;R\ ;z\ ;1)}{@l}\quad\frac{@U_0(l;R\ ;z\ ;0)}{@l} = (\ l\ + (1\quad))\,(cz\quad\frac{1}{2}r) + (1\quad\frac{1}{p})(1\quad l)\ cz$$

$$> 0$$

Immediately, we have

$$\frac{@z}{@l} > 0:$$

Proof of Proposition 7 Optimal Liquidity Holding.

proof

$$\frac{@}{@l}U_0(l; R \ ; z; 1) = \quad (1 \quad c)zx^H + \frac{1}{2}lr + \frac{(1 \quad c)z}{p}(x^H$$

It is negative when $< \frac{z\ \ r}{z\ \ r+(\frac{1}{p}\ \ 1)x_0^L z}$:

The optimal liquidity holding can be positive, $l\ > 0$ if is large and p is small. Note that when $l = 1$, the bank always defaults. So the optimal liquidity holding is bounded above, $l\ < 1$.

When is sufficiently small, $l\ = 0$ for all p.

Proof of Lemma 6 The effect of liquidity regulation on risk-taking z :

Taking derivative of z with respect to ,

$$\frac{@z}{@} = \frac{\frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@}}{\frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@z} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@z}}: \tag{56}$$

Similar to the previous proof,

$$\frac{@z}{@} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@} \quad > 0:$$

where

$$\begin{aligned} &\frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@} \\ = &\ \frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@l} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@l} \quad \frac{@l(\ )}{@} \\ + &\ \frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@R} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@R} \quad \frac{@R}{@l}\frac{@l(\ )}{@}: \end{aligned} \tag{57}$$

With sufficiently low, the liquidity requirement always has binding power,

$$\frac{@l(\ )}{@} = 1:$$

(i) For $p > 1$ There is no defaults induced by liquidity shortage. The illiquidity risk is zero. So the liquid asset holdings do not affect the credit risk, or R .

$$\frac{@R}{@l} = 0:$$

So as proved in Lemma 4,

$$\frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@} = \frac{@V_\theta(l(\ );R\ ;z\ ;1)}{@l} \quad \frac{@V_\theta(l(\ );R\ ;z\ ;0)}{@l} > 0 \tag{58}$$

Therefore, z is increasing in .

(ii) For p 1 Substituting this into Equation 57,

$$\frac{@U_o(I(\ ); R \ ; z \ ; 1)}{@ } \quad \frac{@U_o(I(\ ); R \ ; z \ ; 0)}{@ } = \frac{@U_o(I(\ ); R \ ; z \ ; 1)}{@I} \quad \frac{@U_o(I(\ ); R \ ; z \ ; 0)}{@I}$$

$$+ \quad \frac{@U_o(I(\ ); R \ ; z \ ; 1)}{@R} \quad \frac{@U_o(I(\ ); R \ ; z \ ; 0)}{@R} \quad \frac{@R}{@I}: \qquad (59)$$

Here I show that the expression is negative by approximating $x_0^H \quad x_0^L \quad p$.

From previous analysis, the rst term is given by

$$\frac{@U_o(I(\ ); R \ ; z \ ; 1)}{@I} \quad \frac{@U_o(I(\ ); R \ ; z \ ; 0)}{@I} = (\quad + (1 \quad)) (cz \quad \frac{1}{2} r)$$

$$+ \quad (1 \quad) z \ (x_0^L \quad (1 \quad c) x_0^H \ )(1 \quad \frac{1}{p})$$

$$(\quad + (1 \quad)) (cz \quad \frac{1}{2} r) r$$

So at $p = 0$, the rst term becomes

$$
\begin{aligned}
&\frac{\partial U_\theta(I(\ );R\ ;z\ ;1)}{\partial I} \quad \frac{\partial U_\theta(I(\ );R\ ;z\ ;0)}{\partial I} \\
&= (\quad + (1 \quad))\,(c\frac{1}{2c}R \quad \frac{1}{2}r) + \quad (1 \quad)c\frac{1}{2c}R\ (p \quad 1) \\
&= \frac{1}{2}(\quad + (1 \quad))\,(R \quad r) \quad \frac{1}{2}\ (1 \quad)R \\
&= \frac{1}{2}\ \frac{2}{1} \quad + 2 \quad r \quad (1 \quad)r \quad \frac{}{1}(1 \quad)
\end{aligned} \tag{65}
$$

and it's increasing in : The second term becomes,

$$
\begin{aligned}
&\frac{\partial U_\theta(I(\ );R\ ;z\ ;1)}{\partial R} \quad \frac{\partial U_\theta(I(\ );R\ ;z\ ;0)}{\partial R} \quad \frac{\partial R}{\partial I} \\
&= \quad \frac{1}{2}(1 \quad)\frac{R}{+\ 1} \\
&= \quad \frac{}{+\ 1}
\end{aligned} \tag{66}
$$

and it is decreasing in .

For satis es that $(2 \quad (1 \quad)r)(1 \quad) < 4$, there exists $\ \in (0;1)$, such that for $< \ $,

$$\frac{\partial U_\theta(I(\ );R\ ;z\ ;1)}{\partial} \quad \frac{\partial U_\theta(I(\ );R\ ;z\ ;0)}{\partial} < 0;$$

or z is decreasing in .