ALGEBRA QUALIFYING EXAM FALL 2018

Exercise 1. Suppose p is a prime. Show that the Galois group of x^5 1 2 $\mathrm{F}_p[x]$ depends only on p (mod 5), and compute it for each congruence class of p (mod 5).

Exercise 2. Let R be a Dedekind domain with eld of fractions K Show that for any two proper fractional ideals $I; J$ there are ; 2 K with I; J R integral and I + $J = R$.

Exercise 3. Suppose that R is a Noetherian ring and $\mathfrak{p}$ R is a prime ideal such that $R_{\mathfrak{p}}$ is an integral domain. Show that there is an f 2 R n $\mathfrak{p}$ such that R_f is an integral domain where R_f