Dynamic Team Contests with Complementary Efforts

Maria Arbatskaya and Hideo Konishi

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Abstract

In this paper, we study dynamic team contests. In the framework of a Tullock contest between two teams generating impacts according to the Cobb-Douglas effort aggregation function, we examine how equilibrium

1 Introduction

> "Timing is most important...If the timing is right, even a small action will produce a huge impact. If the timing is wrong, even if you push hard, only little will happen. How do you arrive at the timing? This is a complex affair."
> – Sadhguru

In this paper, we study the effects of timing of the moves by players in team contests. Team contests provide a useful framework for modeling a variety of competitions between firms, political parties, legal teams, academic teams, and sports teams. In a team contest, each player performs a separate task, and the players' efforts are aggregated into a total team impact that, together with other teams' impacts, determines the chances of winning.

We compare different orders of moves by players in team contests. Suppose player 1 on team 1 can commit to an effort and make it known publicly (to his/her team members and to the members of the competing team). Would the player find it beneficial to commit to a higher or lower effort? How does the possibility of such commitment affect all the players' equilibrium efforts and the teams' chances of winning?

To answer these questions, we employ the model developed by a recent paper by Lu and Lu (2020) who consider a two-team contest, in which each team of which is composed of two players assigned to different tasks. Players differ in their marginal costs of making effort. The efforts of players on a team are aggregated by the same Cobb-Douglas function to produce an impact that determines the team's chances of winning through Tullock's contest success function.[1] With this model, Lu and Lu (2020) considered two different scenarios: (i) all players make efforts simultaneously, and (ii) players make efforts in two stages - in the first stage, players who are assigned to the first task choose efforts; then, after observing stage 1 efforts, players assigned to the second task choose their

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[1]

effort levels. Lu and Lu (2020) showed that players' effort levels and winning probabilities are the same between (i) and (ii), that is, equilibrium outcomes in one-stage and two-stage synchronous contests are the same. Building on their model, we analyze a general two-stage contest with an arbitrary order of player moves. To find the equilibrium, we quantify the impacts of each player's effort on the efforts of other players using a powerful elasticity formula. With this tool, we demonstrate in Proposition 1 how the equivalence result in Lu and Lu (2020) holds in synchronous contests.[2]

4, we show that their equivalence results for one-stage and two-stage contests hold for a class of synchronous contests. In Section 5, we show that this is generally not the case in contests where some actions are asynchronous. When information about prior choices is publicly available, the contests are generally unbalancing. In Section 6, we compare all the contests in terms of equilibrium chances of winning. Section 7 concludes the paper.

2 Literature Review

The literature on dynamic contests is extensive. In a pioneering work, Dixit (1987) considered a strategic timing choice game in a contest played by a favorite and underdog, showing that they choose to move sequentially in equilibrium. Extending Dixit's model by allowing for two rounds of effort decisions, Yilidrim (2005) showed that there are multiple subgame-perfect equilibria, while there is no Stackelberg outcome where the underdog leads and the favorite follows. Ludwig (2012) introduced asymmetric information into Dixit's model, analyzing players' timing of moves. In a model with multiple rounds of play, Harris and Vickers (1987) considered an R&D race, analyzing how an initial lead by a team affects the subsequent race. Klumpp and Polborn (2006) asked the same question in the context of the US presidential primary races using the Tullock contests, and Konrad and Kovenock (2008) analyzed the dynamic race more generally using all-pay auctions. All of these studies found that the race favors the player who gained a lead in the initial stages. In contrast, Klumpp, Konrad, and Solomon (2019) showed a time-invariance result in a majoritarian Colonel Blotto problem of allocating a given amount of resources to a finite number of battlegrounds. Each player decides how much resources to spend in the next battleground after each battle's result has been revealed. They show that irrespective of the results of the previous battles, the optimal strategy is to spend the same amount of resources.

efforts are observable by later players. In this model, he found that the total effort is the highest under a fully sequential contest. In contrast, we have teams composed of multiple players; in our team contests, the amount of rent dissipation is the highest in the simultaneous-move contest. Contests played by teams that are composed of multiple players are subject to free-riding among team members. Häfner (2012) considered a tug of war race, which may be played by possibly an infinite number of players, and showed that in his model, there is a unique Markov perfect equilibria. Esteban and Ray (2001) was the first paper that analyzed team contests formally. Assuming that team members' efforts are perfectly substitutable, the authors showed the conditions under which the winning probability of a team increases in its size, despite free-riding incentives. Epstein and Meelem (2009) and Nitzan and Ueda (2011) employed CES effort aggregator functions for team efforts to describe effort complementarities within teams and constant elasticity individual effort costs; they identified the conditions for free-riding incentives to be overcome by effort complementarities. However, these papers are not analyzing dynamic intra- nor inter-team strategic interactions, unlike Lu and Lu (2020) and our paper.

The main result from Lu and Lu (2020) is that the order in which tasks are performed in team contests does not change the equilibrium efforts as long as tasks are chosen synchronously. In contrast, in multi-activity contests among individual players, which in our framework corresponds to teams composed of a single player who chooses efforts in each task, Arbatskaya and Mialon (2012)

3 Model

Consider a contest among two teams, , each having two members responsible for performing task . We will refer to player of team as player . Team members independently choose their effort levels $_{ij}$; and . Players' efforts contribute to their team's chances of winning a prize, which is the public good with a value normalized to . Team members' efforts are aggregated using the Cobb-Douglas function

$$_{i} \qquad _{i1}^{1} \; _{i2}^{2} \tag{1}$$

where $_{j}$

chance of winning, $_1$ $_2$. For future reference, let team 1's relative cost advantage be $_1$ $\left(\frac{c_{21}}{c_{11}}\right)^{1}\left(\frac{c_{22}}{c_{12}}\right)^{2}$.

3.1 Elasticity Representation of the First-Order Conditions

In our analysis, it is convenient to use first-order conditions in terms of elasticities. Let $_{\Theta;e}$ $\frac{d\Theta}{de}\frac{e}{\Theta}$ be the elasticity of team 's (relative) power $_i$ with respect to effort $_{ij}$. The capitalized $_{\Theta;e}$ signifies the total elasticity, where we are taking the total (not partial) derivative: $_{\Theta;e}$ $\frac{d\Theta}{de}\frac{e}{\Theta}$: that is, player evaluates the effect of her effort choice $_{ij}$ by taking the reactions by successive movers (followers) into account. Thus, the first-order condition for player 's effort in an elasticity form is as follows:

$$\frac{_{ij}}{_{ij}} \quad _{\Theta;e} \quad \frac{}{_{ij}} \quad _{ij} \tag{6}$$

or

$$_{\Theta;e} \quad _{ij} \; _{ij} \tag{7}$$

The expenditures (cost of effort) by player are equal to the total elasticity of team 's (relative) power with respect to effort $_{ij}$ times the balance of power. The expenditures $_{ij}$ $_{ij}$ are higher when the contest is more balanced and when the power of team is more responsive to changes in task .

To analyze players' strategic actions, we investigate $_{\Theta;e}$ and in contests with different orders of moves. Thanks to the Cobb-Douglas specification of the effort aggregator function, the elasticity of team 's (relative) power $_i$ with respect to effort $_{ij}$ in partial differentiation is $_{\Theta;e}$ $\frac{\partial\Theta}{\partial e}\frac{e}{\Theta}$ $_j$. Notice that for stage 2 efforts, $_{\Theta;e}$ $_{\Theta;e}$ always holds, but this equality is generally not true for player , who moves in stage 1 if the players in stage 2 can observe $_{ij}$ before they choose their actions. In the following lemma, we list useful elasticity formulas that will be used to simplify the analysis of the strategic responses in the contest.

Lemma 1. Suppose and are ^{1} functions and that is a constant. Then, we have the following:

1. for ^{c}, we have $_{z(x);x}$
2. for , we have $_{z(x);x} \quad _{g(x);x}$
3. for , we have $_{z(x);x} \quad _{f(x);x} \quad _{g(x);x}$
4. for $\frac{f(x)}{g(x)}$, we have $_{z(x);x} \quad _{f(x);x} \quad _{g(x);x}$
5. for , we have $_{z(x);x} \quad _{f(y);y} \; _{g(x);x}$
6. for ^{c}, we have $_{z(x);x} \quad _{g(x);x}$

For example, consider the contest where player 11 moves first by choosing effort $_{11}$ in stage 1. Then, after observing $_{11}$, other players choose their effort levels simultaneously in stage 2: $_{ij} \; _{11}$ for .

To find $_{\Theta_1;e_{11}}$, recall that $_1 \quad \frac{X_1}{X_2} \quad \left(\frac{e_{11}}{e_{21}}\right)^{1} \left(\frac{e_{12}}{e_{22}}\right)^{2}$ and notice that $_1$ is a function of $_{11}$, $_{12}$, $_{21}$, and $_{22}$ with $_{\Theta_1;e_{11}} \quad _1$, $_{\Theta_1;e_{12}} \quad _2$, $_{\Theta_1;e_{21}} \quad _1$, and $_{\Theta_1;e_{22}} \quad _2$. Denoting the elasticity of responses by the followers to $_{11}$ with $_{e_{12};e_{11}}$, $_{e_{21};e_{11}}$, and $_{e_{22};e_{11}}$, we find the following:

$$_{\Theta_1;e_{11}} \quad 1 \quad 2 \; _{e_{12};e_{11}} \quad 1 \; _{e_{21};e_{11}} \quad 2 \; _{e_{22};e_{11}} \tag{8}$$

The responses of the followers depend on the extent of the change in the balance of power, , that player 11's effort brings about. Recall that $_1 \quad _1 \; ^2$, so

$$_{\Lambda;e_{11}} \quad _{\Lambda;\Theta_1} \; _{\Theta_1;e_{11}} \tag{9}$$

where

$$_{\Lambda;\Theta_1} \quad \frac{1}{1} \tag{10}$$

and $_{\Theta_1;e_{11}}$ is stated in equation (8); $_{\Lambda;\Theta_1}$ for $_1$.

In the rest of the paper, we consider this team contest with different timing of moves. Our elasticity formula proves useful in understanding players' strategic incentives. In the next section, we review the equivalence result found in Lu and Lu (2020).

4 Equivalence Result (Lu and Lu 2020)

Lu and Lu (2020) compared the simultaneous move contest (all players choose their effort levels simultaneously) and a synchronous task two-stage contest (task 1's efforts are selected by both teams in stage 1, and task 2's efforts are selected by both teams in stage 2). Somewhat surprisingly, they showed that with a Cobb-Douglas aggregator function, the outcomes of the contests are equivalent, even if the teams and players are asymmetric.

4.1 One-Stage Contest

Let's start with the simultaneous move game. When all efforts are chosen simultaneously, we have $_{X\ ;e}$ $_{X\ ;e}$ $_{j}$. Hence, the first-order conditions (7) can be written as

$$_{ij}\quad _{ij}\quad _{j}$$

where $_{1}$ $_{1}$ ^{2} and $_{1}$ $_{1}$ $_{2}$. This implies $\frac{e_{11}}{e_{21}}$ $\frac{c_{21}}{c_{11}}$ and $\frac{e_{12}}{e_{22}}$ $\frac{c_{22}}{c_{12}}$, and

$$_{1}\quad ^{1}$$

The first-order conditions for stage 1 efforts in elasticity formula are written as

$$i1 \quad i1 \qquad \Theta \; ;e_{1}$$

$$\left(1 \qquad 2 \;\; e_{2};e_{1} \qquad 2 \;\; e_{2};e_{1} \right)$$

$$\left(1 \qquad 2 \;\; \frac{2}{2};e_{1} \right) \qquad 1$$

where ^{0}. The last two equations hold by Lemma 1.4 and $\frac{2}{2};e_{1}$

From, $\Theta;e$ $_{ij}$ $_{ij}$ for , where $_1$ $_1$ ^{2}. In stage 2, after observing $_{11}$, the rest of the players choose their efforts levels simultaneously. For each $_{11}$, the optimal stage 2 efforts, the elasticity representation of the first-order conditions (7) for $_{12}$ $_{11}$, $_{21}$ $_{11}$, and $_{22}$ $_{11}$ are:

$$\begin{aligned} &{}_{12}\ {}_{12}\quad {}_{2} \\ &{}_{21}\ {}_{21}\quad {}_{1} \\ &{}_{22}\ {}_{22}\quad {}_{2} \end{aligned} \tag{11}$$

because $_{\Theta_1;e_{12}}$ $_{\Theta_1;e_{12}}$ $_2$, $_{\Theta_2;e_{21}}$ $_{\Theta_2;e_{21}}$ $_1$, and $_{\Theta_2;e_{22}}$ $_{\Theta_2;e_{22}}$ $_2$.

Following the discussions after Lemma 1 (8), player 11's effort $_{11}$ affects team 1's power as follows:

$$\begin{aligned} &{}_{\Theta_1;e_{11}}\quad {}_{\Theta_1;e_{11}}\quad {}_{\Theta_1;e_{12}}\ {}_{e_{12};e_{11}}\quad {}_{\Theta_2;e_{21}}\ {}_{e_{21};e_{11}}\quad {}_{\Theta_2;e_{22}}\ {}_{e_{22};e_{11}} \\ &\qquad {}_1\quad {}_2\ {}_{e_{12};e_{11}}\quad {}_1\ {}_{e_{21};e_{11}}\quad {}_2\ {}_{e_{22};e_{11}} \end{aligned} \tag{12}$$

so the first-order condition for $_{11}$ is

$${}_{11}\ {}_{11}\quad {}_{\Theta_1;e_{11}}\quad {}_1\quad {}_2\ {}_{e_{12};e_{11}}\quad {}_1\ {}_{e_{21};e_{11}}\quad {}_2\ {}_{e_{22};e_{11}} \tag{13}$$

Thanks to the Cobb-Douglas specification, we can find $_{e_{12};e_{11}}$, $_{e_{21};e_{11}}$, and $_{e_{22};e_{11}}$ from (11) by totally differentiating the identities with respect to $_{11}$ (writing in terms of elasticities) and using the elasticity property 2 of Lemma 1:

$$\begin{aligned} &{}_{e_{12};e_{11}}\quad {}_{e_{12};e_{11}}\quad {}_{\Lambda;e_{11}} \\ &{}_{e_{21};e_{11}}\quad {}_{e_{21};e_{11}}\quad {}_{\Lambda;e_{11}} \\ &{}_{e_{22};e_{11}}\quad {}_{e_{22};e_{11}}\quad {}_{\Lambda;e_{11}} \end{aligned}$$

That is, the elasticity of the impact of an increase in $_{11}$ on stage 2 efforts is the same as the elasticity of its impact on the balance of power . This special property is because of the Cobb-Douglas effort aggregator function, and it simplifies the rest of the analysis tremendously.

Because the elasticity representation of (9) is $\Lambda;e_{11} \qquad \Lambda;\Theta_1 \quad \Theta_1;e_{11}$ $\frac{1\ \ \Theta_1}{1+\Theta_1}\ \ \Theta_1;e_{11}$, we have the following:

$$\Theta_1;e_{11} \qquad 1 \qquad 2\ \ e_{12};e_{11} \qquad 1\ \ e_{21};e_{11} \qquad 2\ \ e_{22};e_{11} \tag{14}$$

$$1 \qquad 1\ \ \Lambda;e_{11}$$

$$1 \qquad 1\frac{1}{1}\ \ \Theta_1;e_{11}$$

Solving the above equation for $\Theta_1;e_{11}$, we obtain the following:

$$\Theta_1;e_{11} \qquad 1\left(\qquad 1\frac{1}{1}\right)^{1} \tag{15}$$

Thus the first-order condition for effort $_{11}$ (7) can be written as follows:

$$11\ \ 11 \qquad 1\left(\qquad 1\frac{1}{1}\right)^{1} \tag{16}$$

By the first-order conditions (7), we have the following:

$$\frac{11\ \ 11}{21\ \ 21} \qquad \frac{\Theta_1;e_{11}}{1} \quad \text{and} \quad \frac{12\ \ 12}{22\ \ 22} \tag{17}$$

Since $_1 \quad \left(\frac{e_{11}}{e_{21}}\right)^{1}\left(\frac{e_{12}}{e_{22}}\right)^{2}$, it follows that

$$1 \qquad \left(\frac{\Theta_1;e_{11}\ \ 21}{1\ \ 11}\right)^{1}\left(\frac{22}{12}\right)^{2} \tag{18}$$

$$\left(\qquad 1\frac{1}{1}\right)^{1}\left(\frac{21}{11}\right)^{1}\left(\frac{22}{12}\right)^{2}$$

Thus, we find that the equilibrium power of team 1, $_1 \qquad _1$, is the solution of

$$1\left(\qquad 1\frac{1}{1}\right)^{1} \qquad 1 \tag{19}$$

where $_1 \quad \left(\frac{c_{21}}{c_{11}}\right)^{1}\left(\frac{c_{22}}{c_{12}}\right)^{2}$ is team 1's relative cost advantage.

The following technical lemma allows us to prove that there exists a unique solution $_1$ to (19) for any costs $_{ij}$ and weights $_j$ for ; and to show that if $_1$, then $_1 \qquad _1$; if $_1$, then $_1 \qquad _1$.

Lemma 2. Let $\left(\qquad \frac{1\ \ x}{1+x}\right)^{a}$ with and . Then, $\frac{@f(x;a)}{@x}$; $\frac{@f(x;a)}{@a}$ iff . Equation with any has a unique solution ; if then ; if then ; if then .

Finally, from the first-order conditions, we have $_{11}$ $_{11}$ $_{1}$, $_{12}$ $_{12}$ $_{2}$, $_{21}$ $_{21}$ $_{1}$, and $_{22}$ $_{22}$ $_{2}$, where $\big(\; _{1}{}^{1} \; \Theta_1$

efforts is beneficial for a player because it saves the cost of a stage 2 task. Here, we have each task managed separately by a team player, so the cost-saving motive is absent. The reason for a commitment here is to exploit the best responses of the competitors, which are negatively sloped for the underdog team (strategic substitutes) and positively sloped for the favorite team (strategic complements).

By Lemma 2, $_1$ $_1$ is an increasing function of $_1$, the equilibrium power of team 1, $_1$, is increasing in $_1$. It is also increasing in $_1$ whenever team 1 is the favorite ($_1$). This means that the favorite team achieves a higher power and probability of winning when the task that the favorite commits to is more influential ($_1$ is higher). The opposite is true regarding the underdog team's commitment. The underdog's team commitment diminishes its power and chances of winning, and this is all the more the case when the task the underdog is committing to is more influential. The extent of the unbalancing depends on the parameter values. It is impossible to strategically unbalance a contest that is perfectly balanced based on the costs of effort ($_1$).

So far, we have examined synchronous two-stage contests and an asynchronous two-stage contest where a team leads in one task. In the next section, we study other cases of two-task, two-stage contests. We also compare the odds of winning by team 1 across the two-stage and one-stage contests.

6 Comparing Across Two-Stage Contests

In this section, we examine two-stage contests with two teams and two tasks. Tasks in a two-stage contest are divided into stage 2 and stage 1. Without a loss of generality, assume $_{11}$ is selected in stage 1. Then, there are five contests to analyze:

C0. (a synchronous two-stage contest) efforts in task 1 are selected in stage 1 and efforts in task 2 are selected in stage 2;

C1. one player (player 11) leads, and the rest of the players follow in stage 2;

C2. one player (player 22) follows, and the rest of the players choose efforts in stage 1;

C3. team 1 leads in both tasks; and

C4. teams lead in different tasks: team 1 leads in task 1 and team 2 leads in task 2.

Contests C0 - C4 cover all orders of move in two-stage contests because the results for any other two-stage contests can be obtained by relabeling the teams, tasks, or both. (To switch teams, replace $_1$ with $_2$ $_1$; to switch tasks, switch $_1$ and $_2$.)

For any costs $_{ij}$ and weights $_j$, we can compute and compare the equilibrium efforts ($_{11}$, $_{12}$, $_{21}$, $_{22}$) in two-stage contests and in the one-stage contest. We can also compare the power of team 1 and the associated balance of power across the contests. Figure 1 shows the stage 1 and stage 2 efforts in contests C0 - C4. Proposition 3 summarizers the properties of the equilibria. We show that in general, any asynchronous two-stage contest with publicly observable commitments is strategically unbalancing by the choice of stage 1 tasks, when compared with a synchronous contest C0 (which is equivalent to the one-stage contest).

Proposition 3. For any costs $_{ij}$ and weights $_j$, , there exists a unique subgame-perfect equilibrium in every two-stage contest C0 - C4. The equilibrium power of team 1, $_1$ $_1$, is defined by $_1$ $_1$, with $_1$ $_1\left(\frac{1\quad\Theta_1}{1+\Theta_1}\right)^{a}$, where takes the following values for each case: C0: , C1: $_1$, C2: $_2$, C3: $_1$ $_2$, and C4: $_1$ $_2$; stage 1 efforts are $_{ij}$ $_1$ $\overline{}\,\mathrm{c}$ $_1$, with $_1\left(\frac{1\quad\Theta_1}{1+\Theta_1}\right)^{1}$ iff $_1$; and stage 2 efforts are $_{ij}$ $\overline{}\,\mathrm{c}$ $_1$, where $_1$ $_1$ $_1$ ^{2}. The favorite (1 + (underdog)

The derivations of the subgame-perfect equilibria for contests C2 - C4 follow the same steps as for contest C1, which was analyzed in the previous section and is omitted here. We know that contest C0 is equivalent in terms of outcomes to the one-stage contest, in which all tasks are chosen simultaneously. However, contests C1 - C4 do not show this equivalence result.

There are strategic incentives to change stage 1 tasks, but no such strategic incentives exist for stage 2 tasks. All stage 1 efforts are overcommitted by the same scaling factor , which is the same for all stage 1 tasks but different across contests C1 - C4; stage 1 efforts are scaled up by a factor if team 1 is the favorite ($_1$). All stage 1 efforts are scaled down by a scaling factor if team 1 is the underdog ($_1$). A perfectly balanced contest remains perfectly balanced: $_1$ if $_1$, with no changes in the equilibrium levels of stage 1 tasks.

The unique solution to the equilibrium power of team 1, $_1$, is found numerically. However, the equilibrium properties, such as the over/under commitment by stage 1 players, can be obtained without finding the value for $_1$. Importantly, assuming $_1$, the two-stage contest is always unbalancing (the weak team becomes weaker, and the strong team becomes stronger in two-stage contests). The only exception is in contest C4, where players cross-lead with equally influential tasks, that is, when $_1$ $_2$. In this case, there is no unbalancing, and there is no strategic advantage to team 1.

The following proposition provides some implications about the timing of moves that a team leader would prefer if this leader cared about their team's probability of winning.

Proposition 4. The favorite team has its highest chances of winning when it leads in both tasks and the lowest chances of winning in the synchronous contest. The favorite team has more power when it leads than when it follows (in one task, in both, or in a more influential task). The opposite is true for the underdog team.

We always assume $_1$, $_2$, and $_1$ $_2$. If $_1$ $_2$ and $_1$ $_2$

of moves with publicly observed commitments. The problem is complex because when a player makes a choice he/she needs to consider the influence of his/her choice on the choices of all of his/her team members and rivals. Luckily, with the identical Cobb-Douglas effort aggregator function across teams, some effects cancel out. For example, the effect on a player's teammate and their direct opponent at the next stage cancel out because they change by the same percentage and the ratio of their efforts stays the same. Hence, when there are only two stages and two tasks, a player who can precommit would only have to consider the effect of his/her choice on his/her direct opponent if the opponent moves later. No change occurs when he/she moves together with his/her direct opponent (synchronous moves); even though the player has the ability to change the effort levels of his/her teammate and his/her direct opponent, these changes do not affect his/her payoff, which depends on the ratio of efforts in other tasks. Therefore, the Cobb-Douglas effort aggregator function allows us to cancel out all synchronous moves in other tasks. That is, it is possible to extend our analysis to accommodate the multiple-task case: it is clear that the equivalence result in Lu and Lu (2020) extends to any synchronous temporal structure in multiple task cases, which follows the spirit of Fu, Lu, and Pan (2015). In contrast, without synchronous moves for the same tasks, it is also clear that the equivalence theorem does not hold in general.

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Appendix A: Proofs

$_{12}$ $_{12}$ $_{2}$, $_{21}$ $_{21}$ $_{1}$, $_{22}$ $_{22}$ $_{2}$, where $\Big($ $_{2}{}^{1}$ Θ_1

Similarly, $_{\Theta_2;e_{22}}$ $_2$ $_1$ $_2$ $\frac{1\ \ \Theta_2}{1+\Theta_2}$ $_{\Theta_2;e_{22}}$, and therefore $_{\Theta_2;e_{22}}$ $_2$.

We have $_{11}$ $_{11}$ $_1$, $_{12}$ $_{12}$ $_2$, $_{21}$ $_{21}$ $_1$, and $_{22}$ $_{22}$ $_2$, where $\left(\quad _1 \quad _2 \frac{1\ \ \Theta_1}{1+\Theta_1}\right)^{1}$; $_1$ $\left(\frac{e_{11}}{e_{21}}\right)^{1}\left(\frac{e_{12}}{e_{22}}\right)^{2}$ $_1$ ^{1} ^{2}, where $_1$ $\left(\frac{c_{21}}{c_{11}}\right)^{1}\left(\frac{c_{22}}{c_{12}}\right)^{2}$, and therefore $_1$ $_1$ is a solution to $_1\left(\quad _1 \quad _2 \frac{1\ \ \Theta_1}{1+\Theta_1}\right)^{1 \quad 2}$ $_1$. By Lemma 2, for any costs $_{ij}$ and weights $_j$, , there exists a unique subgame-perfect equilibrium $_1$; if $_1$ then $_1$ $_1$; if $_1$, then $_1$ $_1$; i¤ $_1$ $_2$ $_1$.□

Proof of Proposition 4. By Proposition 3, the equilibrium power of team 1, $_1$ $_1$, is de…ned by $_1$ $_1\left(\quad \frac{1\ \ \Theta_1}{1+\Theta_1}\right)^{a}$ $_1$, where equals , $_1$, $_2$, $_1$ $_2$, and $_1$ $_2$ for contests C0 - C4, correspondingly. Since we assume $_1$, $_2$, and $_1$ $_2$, we have . By Lemma 2, for any $_1$ and , $\frac{@f(\Theta_1;a)}{@\Theta_1}$; $\frac{@f(\Theta_1;a)}{@a}$ i¤ $_1$. Equation $_1$ $_1$ then implies that $\frac{@\Theta_1}{@a}$ if $_1$ $_1$ and $\frac{@\Theta_1}{@a}$ if $_1$ $_1$.

We can then rank the contests C0 - C4 in terms of $_1$ for $_1$. If $_1$ $_2$ and $_1$ $_2$, then $_1$ $_2$ $_1$ $_2$ $_1$ $_2$ and C3 C1 C2 C4 C0. If $_1$